On statistical theories of p-fluids[1]

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Abstract

We consider the 3D Navier-Stokes equations

$$\frac{\partial u}{\partial t} + \nabla \cdot (u \otimes u) + \nabla p = \frac{1}{Re} \triangle u + f, \quad \text{div } u = 0, \quad u|_{t=0} = u_0,$$

describing the motion of an incompressible fluid with constant density. To capture all scales, a number of degrees of freedom proportional to $Re^{9/4}$ is required, as follows from the K41-argument of Kolmogorov [1]. This result implies severe computational restrictions to the numerical simulation of real-life fluids. To overcome this problem, many approximate models have been proposed, aimed at capturing the large scales, but still keeping a reasonable computational cost. One of the most known and used is that proposed by Smagorinsky [4]

$$\frac{\partial \overline{u}}{\partial t} + \nabla \cdot (\overline{u} \otimes \overline{u}) + \nabla \overline{p} = \nabla \cdot \left(\frac{1}{Re} + (C_S \delta)^2 \, |\nabla^s \overline{u}|^{p-2} \right) \nabla \overline{u} + \overline{f} \qquad p = 3,$$

where $\delta > 0$ is the cut-off scale, while $\overline{u}$ is an approximation of u. The Smagorinsky model represent a *paradigm* in the Large Eddy Simulation of turbulent flows. Concerning the estimation of the "universal" constant $C_S > 0$ the first result dates back to Lilly [3]. Here, we use ideas coming from scaling invariance and from stochastic treatment of the Navier-Stokes equations, in order to estimate the constant C_S for different values of p. We propose a new justification of the fact that $\overline{u}$ should be a large scale approximation of u and -based on this justification- we also give heuristics for the use of different values of p, see also Layton [2]. We give an explanation of the over-damping observed in numerical simulations for $p = 3$ and we observe that the over-damping could be less strong if p is large and there is some degree of intermittency.

Keywords: LES, homogeneous turbulence, K41, scaling invariance.

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