Isogeometric Analysis of Fluid-Structure Interaction

Y. Bazilevs, V.M. Calo, T.J.R. Hughes
Institute for Computational Engineering and Sciences, The University of Texas at Austin, USA
e-mail: {bazily,victor,hughes}@ices.utexas.edu

Y. Zhang
Department of Mechanical Engineering, Carnegie Mellon University, USA
jessicaz@andrew.cmu.edu

Abstract

Isogeometric analysis is a recently developed methodology based on technologies that were originated in the field of computational geometry and widely used in design, graphics and animation. It includes standard finite element analysis as a special case, but offers other possibilities that are unique and powerful. It allows more precise and efficient geometric modeling, it simplifies mesh refinement, and it possesses superior approximation properties. Isogeometric analysis has been applied to numerous problems in solid and fluid mechanics. This paper describes an isogeometric formulation for fluid-structure interaction of incompressible fluid flow and nonlinear solids. The fluid discretization derives from a residual-based variational multiscale formulation, applicable to laminar and turbulent phenomena. Both fluid and solid domains may undergo large motions, and the geometry and kinematics are fully compatible across fluid-structure interfaces. A strongly coupled solution algorithm is adopted to preclude instabilities that often afflict weakly-coupled procedures. Applications to the human cardiovascular system are emphasized.

Keywords: Isogeometric Analysis, NURBS, Fluid-Structure Interaction, Vascular Modeling, Navier-Stokes Equations, Elastic Arterial Wall, Mesh Movement, Blood Flow

1 Introduction

Isogeometric Analysis based on NURBS (non-uniform rational B-splines) was first introduced in [1] as an attempt to improve on and generalize the standard finite element method. Further study of isogeometric analysis showed that results superior to standard finite elements are obtained in the context of structural vibrations [2]. Mathematical analysis of the isogeometric approach was performed in [3]. Optimal approximation estimates in p, the polynomial order used to define NURBS functions, were obtained for h-refined meshes. Stability and optimal convergence was proved mathematically and verified numerically for problems of compressible and incompressible elasticity, Stokes flow, and scalar advection-diffusion. In this paper, NURBS-based isogeometric analysis is applied to fluid-structure interaction (FSI) problems with particular emphasis on arterial modeling and blood flow (see [4] for a more detailed exposition). It is believed that the ability of NURBS to accurately represent smooth exact geometries, that are natural for arterial systems, but unattainable in the faceted finite-element representation, and the high order of approximation of NURBS, should render fluid and structural computations more physiologically realistic.

This work adopts the arbitrary Lagrangian-Eulerian (ALE) framework. The arterial wall is treated as a nonlinear elastic solid in the Lagrangian description governed by the equations of elastodynamics. Blood is assumed to be a Newtonian viscous fluid governed by the incompressible Navier-Stokes equations written in the ALE form. The fluid velocity is set equal to the velocity of the solid at the fluid-solid interface. The coupled FSI problem is written in a variational form such that the stress compatibility condition at the fluid-solid interface is enforced weakly. The ALE equations require the specification of the fluid region motion. This motion is found by solving an auxiliary static linear-elastic boundary-value problem for which the fluid-solid boundary displacement acts as a Dirichlet boundary condition.

Galerkin's method is employed for the structural and the fluid subdomain motion parts of the formulation, while a residual-based multiscale method for the fluid equations. The resultant semi-discrete equations are advanced in time using the generalized-α algorithm. The kinematic constraint is enforced strongly by requiring basis functions to be C^0-continuous across the fluid-solid interface. The coupled nonlinear system resulting from the NURBS discretization of the FSI equations is solved monolithically, that is, the fluid, the structural, and the mesh solution increments in the Newton iteration are obtained simultaneously. The effect of the structural and the mesh motion on the fluid equations is included in the left-hand-side matrix for robustness. The coupled system is solved iteratively by the GMRES procedure with simple diagonal scaling.

2 Numerical examples

2.1 Blood flow in an idealized aneurysm

In this test case, taken from [5], we examine pulsatile flow in an idealized aneurysm. The problem setup is shown in Figure 1. A time-periodic velocity waveform, specified at the inflow plane, is parabolically distributed over the circular surface. The domains proximal and distal to the aneurysm region are assumed to have rigid walls, while the aneurysm wall is elastic. A resistance boundary condition is applied at the outflow.

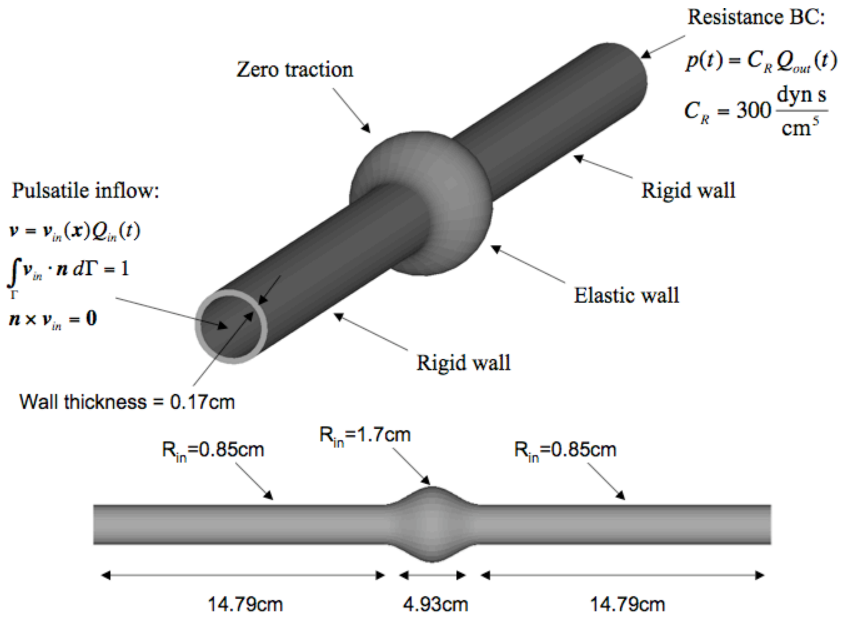


Figure 1: Idealized aneurysm problem setup.

Figure 2 shows the inflow and outflow waveforms. Note the outflow lags the inflow due to the distensibility of the aneurysm wall. This well-known phenomenon was also observed in practice as well as computations of other researchers. Figure 2 also shows excellent agreement with reference results of [5].

2.2 Blood flow in a patient-specific abdominal aorta

This computational example makes use of patient-specific geometry. Data for this model was obtained from 64-slice CT angiography of a healthy male over 55 years of age. Some preprocessing, including contrast enhancement, denoising, and segmentation, was necessary in order to find the lumenal surface of the blood vessel. Figures 3(a) - 3(c) show the geometrical model, the control mesh, and the NURBS mesh of the abdominal aorta. As in the previous example, we specify a time-periodic velocity waveform at the inflow, while all outflows are assigned a resistance boundary condition. Figure 3(d) shows contours of the arterial wall velocity magnitude plotted on a deformed configuration during early systole.

References

[1] T.J.R. Hughes, J.A. Cottrell, and Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Computer Methods in Applied Mechanics and Engineering*, **194** (2005) 4135-4195.

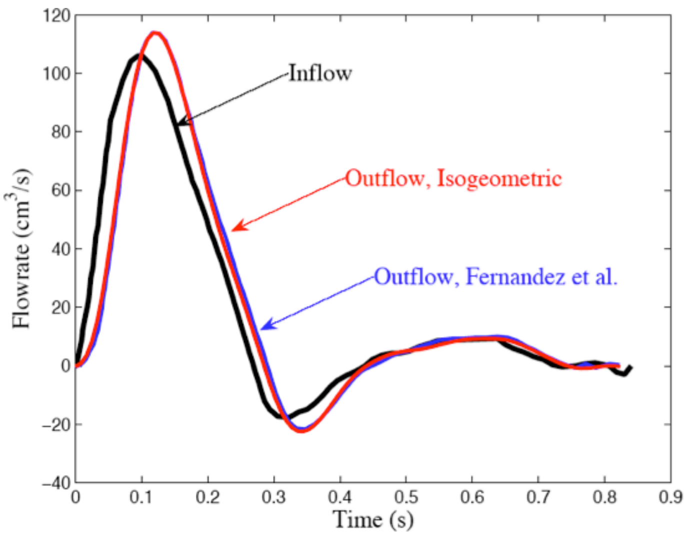


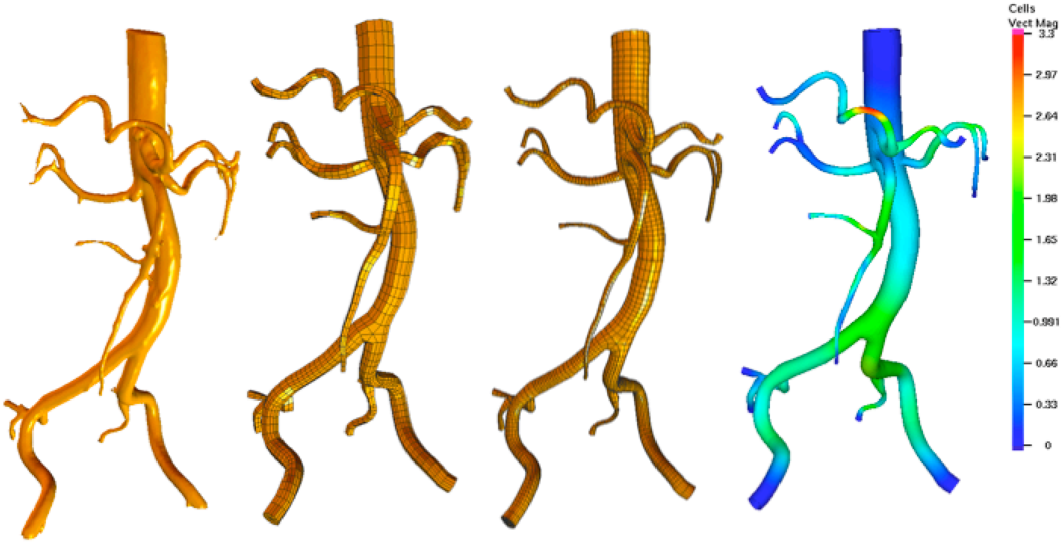
Figure 2: Idealized aneurysm. Inflow and outflow waveforms. Notice the time lag attributable to the distensibility of the wall.

[2] J.A. Cottrell, A. Reali, Y. Bazilevs, and T.J.R. Hughes. Isogeometric analysis of structural vibrations. *Computer Methods in Applied Mechanics and Engineering*, **195** (2006) 5257-5296.

[3] Y. Bazilevs, L. Beirao da Veiga, J.A. Cottrell, T.J.R. Hughes, and G. Sangalli. Isogeometric analysis: Approximation, stability and error estimates for h-refined meshes. *Mathematical Methods and Models in Applied Sciences*, **16** (2006) 1031-1090.

[4] Y. Bazilevs, V.M. Calo, Y. Zhang, and T.J.R. Hughes. Isogeometric fluid-structure interaction analysis with applications to arterial blood flow. *Computational Mechanics*, **38** (2006) 310-322.

[5] A.-V. Salsac, M.A. Fernandez, J.-M. Chomaz, and P. Le Tallec. Effects of the flexibility of the arterial wall on the wall shear stress and wall tension in abdominal aortic aneurysms. *Proceedings of the 58th Annual Meeting of the Division of Fluid Dynamics*, Chicago, IL, November 2005.



(a) (b) (c) (d)

Figure 3: Patient-specific abdominal aorta. (a) Geometrical model; (b) Control mesh; (c) NURBS mesh; (d) Contours of arterial wall velocity magnitude plotted on a current configuration during early systole.