Geometrical Multiscale Modelling of the Integrated Cardiovascular System

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Abstract

Over the past years, mathematical modelling and numerical simulation play a very important role in the understanding of the cardiovascular system, helping in the prediction of the origin and the development of pathologies, as well as the result of surgical interventions. Nevertheless, the functional and geometrical complexity of the human circulatory system make it a very difficult and challenging task to model. For instance, three-dimensional (3D) simulations are restricted to truncated regions of interest, due to their computational costs and the impossibility of having a 3D representation of the whole cardiovascular system. However, local realistic simulations can not be carried out without taking into account the global circulation, which is commonly neglected. Also, blood flow is a non-Newtonian fluid, presenting a shear-thinning behaviour [1], which should be taken into account, specially in small vessels or certain pathological cases. The geometrical multiscale modelling consists in coupling together different models with different levels of complexity and detail. In this way, the most adequate model can be applied to each part of the cardiovascular system, or type of investigation at hand, in an integrated manner. In this context, the remaining parts of the cardiovascular system in a local 3D simulation can be taken into account resorting to reduced one-dimensional (1D) or lumped parameters (also called zero-dimensional, 0D) models, Figure 1.

We present the geometrical multiscale approach. We describe the different models, from 3D fluid-structure interaction (FSI) and generalized Newtonian models, to the 1D hyperbolic model, applied to study the pressure wave propagation in large networks of arteries, and lumped parameters models representing wide compartments, such as the heart or the resistance due to the venous bed. The different mathematical nature of all these models makes their integration the main challenge of this approach [3, 4, 5]. We discuss the techniques and strategies to couple them, both at the analytical and numerical levels, evidencing the main difficulties. In particular, the stability of the coupling between the 1D and the 3D FSI models is guaranteed through a non standard curl formulation of the 3D fluid equations. The effectiveness of the couplings and the geometrical multiscale approach is illustrated through numerical results.

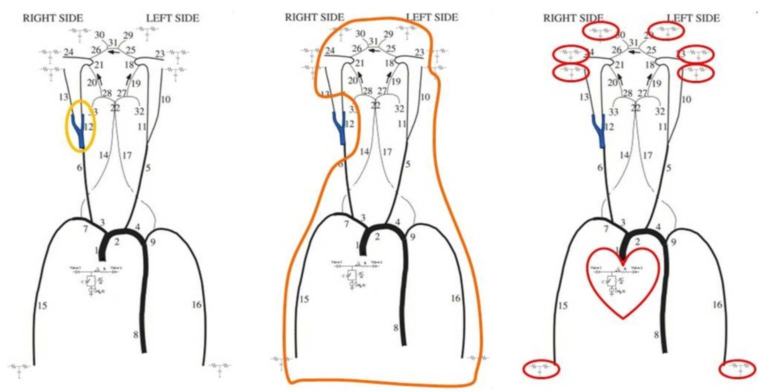

Figure 1: *Scheme of a geometrical multiscale model: coupling between a 3D carotid bifurcation (left), a 1d network representing the arteries from the heart to the Circle of Willis (middle) and a lumped parameters model representing the heart and terminal resistances (right).*

Keywords: Blood flow simulations, geometrical multiscale modelling, coupling strategies, fluid-structure interaction (FSI), hemorheology, 3D FSI model, reduced 1D and 0D models.

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