Convective dynamo in a rotating plane layer

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Abstract

Magnetic fields of planets and other astrophysical objects are often sustained by conducting fluid motions, driven by convection in their interior. A common feature of such objects is rotation. This work is aimed at investigation of magnetic field generation in an idealized setup. We consider a rotating conducting fluid heated from below in a plane horizontal layer (often regarded as representing a segment of a spherical shell in the interior of a planet). Flows and magnetic fields in square periodicity cells are examined for the aspect ratio, for which the trivial hydrodynamic steady state becomes unstable to square patterns at the minimal Rayleigh number.

Thermal convection of electrically conducting fluid in a plane horizontal layer is considered here in the Boussinesq approximation. The fluid is heated from below in a plane horizontal layer rotating about the vertical axis, the stress-free isothermal horizontal boundaries being perfect electrical conductors. In the dimensionless form the CHM system is characterised by the Rayleigh (Ra, measuring the amplitude of buoyancy forces), Prandtl (P, the ratio of kinematic viscosity to thermal diffusivity), magnetic Prandtl (P_m, the ratio of kinematic viscosity to magnetic diffusivity) and Taylor (Ta, measuring the rate of rotation) numbers. We investigate numerically the influence of rotation on the dynamo properties of the convective fluid flows by examining the structure of the hydrodynamic and CHM attractors for different values of Ta, and by considering the kinematic dynamo problem for the hydrodynamic attractors. The existence regions (in Ta) of the attractors of different geometry have been explored applying the technique of continuation in the parameter.

Computations have been performed in square periodicity boxes of size $L = 2\sqrt{2}$ in a layer of depth 1 for $Ra = 2300$, $P = 1$ and $P_m = 8$. The assumed spatial period corresponds to the horizontal wavenumber $k = \pi/\sqrt{2}$ of the first unstable Fourier mode (in the absence of magnetic field) for stress-free boundaries [1]. For this particular Rayleigh

number $Ra = 2300$, in the absence of rotation the hydrodynamic attractor is a travelling wave. This flow is a kinematic dynamo for $P_m > 5.48$ [2]. In our simulations Taylor number varied from $Ta = 0$ (no rotation) to $Ta = 2000$ (with the Coriolis force suppressing the fluid motion).

The symmetry group of the convective hydromagnetic system is $\mathbf{Z}_4 \times T^2 \times \mathbf{Z}_2 \times \mathbf{Z}_2$, where $\mathbf{Z}_4$ is generated by rotation by $\pi/2$ about the vertical axis, the subgroups T are translations in horizontal directions, one $\mathbf{Z}_2$ is reflection about the horizontal midplane and the other one stems from magnetic field reversal. Attractors have been classified in particular by their symmetries, and bifurcations occurring in the system have been identified in terms of symmetry breaking.

For parameter values under consideration hydrodynamic attractors (in the absence of magnetic field) of the convective system are steady rolls or travelling waves. For such flows the kinematic dynamo problem can be reduced to an eigenvalue problem; it has been solved using the algorithm [3].

The problem was studied numerically applying standard pseudospectral methods [4]. Most calculations were performed with the spatial resolution of $64 \times 64 \times 33$ Fourier harmonics. For certain Ta the computations were redone with the resolution of $128 \times 128 \times 65$ harmonics to check the accuracy; the results remain qualitatively unaffected. Integration in time is done by a variant of the Adams-Bashforth method which reduces the stiffness of the system.

In all the saturated nonlinear regimes that we have obtained, the flow has the structure of deformed rolls, magnetic energy remains much smaller than the flow kinetic energy, and magnetic field (when generated) is concentrated near the horizontal boundaries in half-ropes, each spread along the flow streamlines (due to the flow advection) and cut into halves by the horizontal boundary along the rope axis.

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