Stabilization Methods for Shallow-Water Equations

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Abstract

We consider streamline diffusion, also known as SUPG (Streamline Upwind Petrov–Galerkin), methods applied to the time-dependent shallow-water equations. Streamline diffusion (SD) methods are finite element methods that combine good stability properties with high accuracy and are particularly suitable for hyperbolic and advection-diffusion equations. The SUPG method, introduced by Thomas Hughes and Alexander Brooks in 1979 [5], was applied and analysed intensively throughout the 80's by Thomas Hughes and, in parallel, by Claes Johnson, and their co-workers, see, *e.g.*, [9, 1, 12, 10, 8, 11, 6, 4, 7, 2, 3]. Claes Johnson adopted the name streamline diffusion method [9], extended it to the time–dependent problems and related the method, regarding the time discretization, to the discontinuous Galerkin method [12, 10].

Written in conservation form (mass/momentum flux), the shallow-water equations constitute a non-linear hyperbolic system, similar to the compressible Navier-Stokes equations, and their numerical approximation, either in conservative or non-conservative form, has been obtained by various finite difference and finite element methods, most recently by local discontinuous Galerkin methods. Rigorous error analyses have, however, been scarce and even more so for the fully discretized problem written in terms of the non-conservative variables – the depth-integrated horizontal velocities and the height of the free surface.

In this talk, I will present some of our recent results on the application of SD methods, with time–space elements, to two–dimensional shallow-water equations written in a non–conservative form. We will prove error estimates of order h^k and $h^{k+1/2}$ using a suitably stabilized variational formulation. Our finite element approximation is continuous in space but possibly discontinuous in time and we use k^{th}–order polynomials for the surface height and polynomials of order k or $k+1$ for the velocities.

This is a joint work with Clint Dawson from the Center for Subsurface Modeling at the Institute for Computational Engineering and Sciences at the University of Texas at Austin (USA).

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