Numerical Methods and CFD Techniques for Drug Application in Brain Tumor Treatment

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Abstract

In our work, we research drug delivery to brain tumors. The mathematical model of drug delivery to brain tumors is not new, but with various configurations of different complexity. We considers one particular way of treatment of brain tumors: after surgical removal of a cancer tumor doctors continue treatment by means of anti-cancer medicines. The governing system of equations consists of a mass conservation equation, a momentum equation and an equation for the drug concentration in the brain tumor, we consider full mass conservation and momentum equations which are coupled with the concentration equation depending on the velocity. This system describes the penetration of drugs into the brain tumor (there is a cavity after surgical removal of a cancer tumor), which fill up the cavity after a surgery.

We use techniques of computational fluid dynamics (CFD), which are based on the Finite Element Method with multigrid solvers, to get a solution of the derived partial differential equations (Navier-Stokes equation with additional scalar equations and force terms) and obtain a saddle point problem after discretization of the governing system of equations with finite elements such that we can use modern CFD tools and software like FEATFLOW [6] to get numerical solutions of this problem.

We have obtained a closed system of PDE's (partial differential equations) for pressure, vector of velocity and drug concentration with defined boundary and initial conditions. We non-dimensionalized the given equations, which helped to simplify the given equations. We found out that the saddle-point problem requires techniques for incompressible flow problems and we can solve this CFD problem using numerics methods for CFD.

Keywords: brain tumor, drug delivery, saddle-point problem, finite elements methods.

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