Spatial Distribution of Cell Populations in the Processes of Erythropoiesis

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Abstract

We model erythropoiesis, the process which consists in production of red blood cells (RBC) in the bone marrow. Blood cell production starts with hematopoietic stem cells (HSCs) which differentiate into several cell lineages and among them erythroid lineage. Consecutive differentiation of HSCs leads to appearance of immature erythroid progenitors, that in turn undergo several maturity stages to become mature red blood cells (erythrocytes). At each cell cycle, erythroid progenitors can self-renew, differentiate or die by apoptosis.

We will take here into account spatial cell distribution inside the bone marrow and cell motion resulting from cell proliferation. Immature erythrocytes appear and push each other through the medium formed by other cells and by porous matrix. In the process of this motion, cells increase their maturity. Mature blood cells are pushed out into blood vessels going through the marrow. Thus, normal erythropoiesis implies some spatial cell organization according to their maturity level.

Excessive proliferation of immature cells, which can be related to certain blood diseases including leukemia, changes normal cell distribution in the marrow. If proliferation of malignant cells is sufficiently fast, then the tumor grows and can fill the whole marrow. The propagation of leukemic cells corresponds to travelling wave solutions of reaction-diffusion-convection equations. We will study in this work spatial cell distribution for both normal and leukemic erythropoiesis.

We describe cell concentrations by reaction-diffusion equations and their motion by Darcy's law. A close problem was studied in [1] in the 1D spatial case. We are particularly

interested by propagation of 2D waves which correspond to leukemia development in the bone marrow.

We prove the existence of a stationary solution in the 1D case. This solution gives a stationary cell distribution in the cross-section of the bone marrow considered as a 2D rectangular domain. In the leukemic case, this 1D solution can become unstable. The region filled by malignant cells will propagate and fill the whole domain. We study this phenomenon numerically. We will give an analytical approximation for the speed of the travelling wave and will compare it with the numerical results.

Keywords: cell population, reaction-diffusion equations, porous medium, travelling waves.

References

[1] A. Ducrot and V. Volpert, *On a model of leukemia development with a spatial cell distribution,* Math. Model. Nat. Phenom. 3 (2007), pp. 101–120.