Modelling and numerical simulations of the mechanics of cerebral arterial tissue: structural damage.

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Outline

Arterial wall modelling

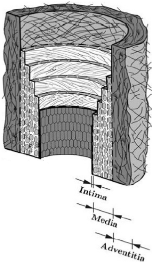

Figure: Structure of healthy arterial tissue

real	modelled
nonlinear behaviour	nonlinear elastic models
layered structure	unique layer
nonhomogeneous and anisotropic	homogeneous and isotropic

Mathematical description the healthy arterial tissue

Cauchy Hyperelastic Isotropic material

$$\Downarrow$$

$$\mathcal{W}(\mathbf{C}) = \mathcal{W}(I_{\mathbf{C}}, II_{\mathbf{C}}, III_{\mathbf{C}})$$

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In this work:

Multiplicative decomposition of **F** and **C**[a]:

$$\mathbf{F} = (J^{1/3}\mathbf{I})\overline{\mathbf{F}} \quad \mathbf{C} = (J^{2/3}\mathbf{I})\overline{\mathbf{C}}$$

[a] Holzapfel - *Nonlinear Solid Mechanics*, Wiley, 2000 and references therein.

Compressible decoupled form of the strain energy function

$$\Downarrow$$

$$\mathcal{W}(I_{\bar{\mathbf{C}}}, II_{\bar{\mathbf{C}}}, J) = \overline{\mathcal{W}}(I_{\bar{\mathbf{C}}}, II_{\bar{\mathbf{C}}}) + \widetilde{\mathcal{W}}(J)$$

Constitutive relations

Isochoric Part

- First order exponential law:

$$\overline{\mathcal{W}}(I_{\bar{\mathbf{C}}}) = \frac{\eta}{\gamma}\left(e^{\gamma(I_{\bar{\mathbf{C}}}-3)} - 1\right),$$

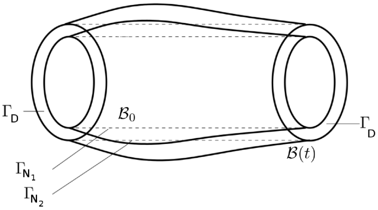
- St. Venant-Kirchhoff law:

$$\overline{\mathcal{W}}(I_{\bar{\mathbf{C}}}, II_{\bar{\mathbf{C}}}) = \left(\frac{\lambda}{8} + \frac{\mu}{4}\right) I_{\bar{\mathbf{C}}}^2 - \left(\frac{3}{4}\lambda + \frac{\mu}{2}\right) I_{\bar{\mathbf{C}}} - \frac{\mu}{2} II_{\bar{\mathbf{C}}} + \frac{9}{8}\lambda + \frac{3}{4}\mu.$$

Volumetric Part

$$\widetilde{\mathcal{W}}(J) = \frac{\kappa}{2}\left[(J-1)^2 + \ln^2(J)\right].$$

The mathematical problem



$$\begin{cases} \rho_s \dfrac{\partial^2 \underline{\eta}}{\partial t^2} = \rho_s \underline{b} + \mathrm{Div}(\mathbf{P}) & \text{in} \quad \mathcal{B}_0 \vee t > 0, \\ \underline{\eta}(0) = \underline{\eta}_0 & \text{in} \quad \mathcal{B}_0 \vee t = 0, \\ \underline{v}(0) = \underline{v}_0 & \text{in} \quad \mathcal{B}_0 \vee t = 0, \\ \underline{\eta}(t) = \underline{0} & \text{on} \quad \Gamma_D, \quad t > 0, \\ \mathbf{P}\underline{n}_1 = \underline{0} & \text{on} \quad \Gamma_{N_1}, \quad t > 0, \\ \mathbf{P}\underline{n}_2 = \underline{t}_0 & \text{on} \quad \Gamma_{N_2}, \quad t > 0, \end{cases}$$

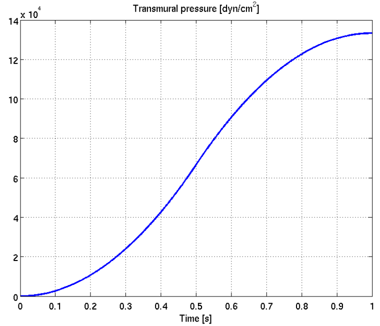
where **P** is the first Piola-Kirchhoff tensor, $\mathbf{P} = \bar{\mathbf{P}} + \tilde{\mathbf{P}} = 2\mathbf{F}\left[\dfrac{\partial \bar{\mathcal{W}}}{\partial \mathbf{C}} + \dfrac{\tilde{\mathcal{W}}}{\partial \mathbf{C}}\right]$

Problem setting

Initial conditions:

- $\underline{\eta}_0 = 0$ on $\mathcal{B}_0$
- $\underline{v}_0 = 0$ on $\mathcal{B}_0$

Load boundary conditions:



Material parameters:
Exponential law:

- $\eta = 1.700e + 6$ dyn/cm^2
- $\gamma = 0.6$

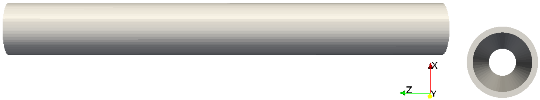
St. Venant-Kirchhoff law:

- $E = 5e + 6$ dyn/cm^2
- $\nu = 0.47$

Space and Time discretization:

- $\mathbb{P}1$ Finite Element Space;
- Newmark method;

Geometry and Problem Dimensions



(a) (b)

Figure: Computational domain.

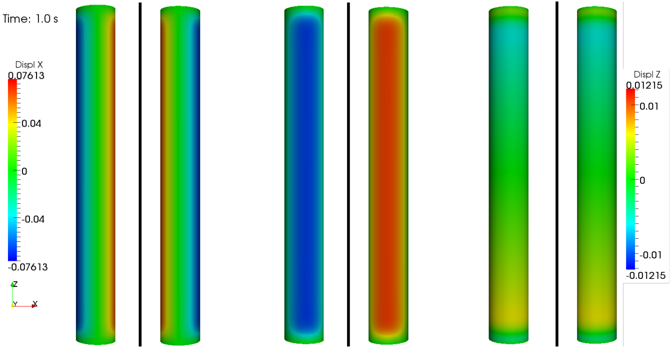
Geometry:

- $L = 10$ cm
- $D = 1$ cm
- $H = 0.1$ cm

Discretization:

- h = 0.01 cm
- # tetrahedra = 399840
- # DOFs = 229824

Numerical Simulations: inflation of healthy artery

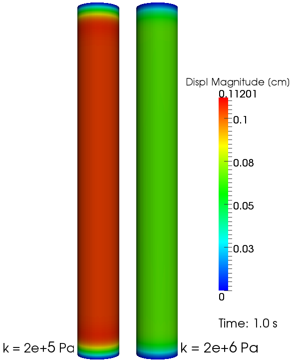


(a) *x*-component (b) *y*-component (c) *z*-component

Figure: Displacement field at the transmural pressure of 13.3 kPa

As expected, the deformation along the axis are symmetric.

Inflation of healthy artery - The role of the bulk modulus



(a) Displacement Magnitude

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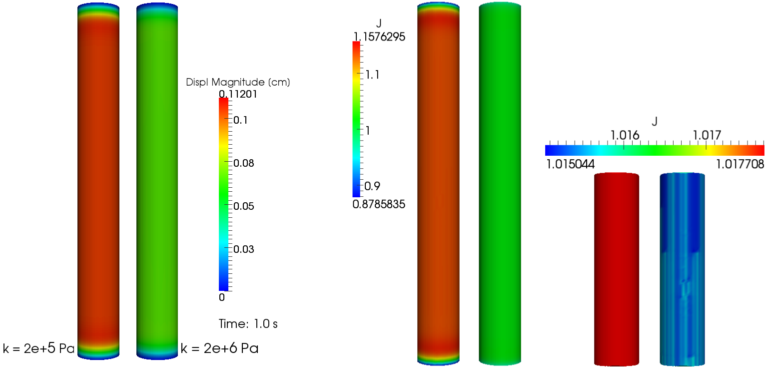

(a) Displacement Magnitude (b) J = det(F) (c) Central Portion of the domain

Figure: Displacement field and distribution of J = det(**F**) at the transmural pressure of 13.3 kPa

Higher bulk moduli lead to a better approximation of the volume-preserving constraint at higher computational cost.

Inflation of a weakened artery

Healhty artery

$$\overline{\mathcal{W}}(I_{\bar{\mathbf{c}}}) = \frac{\eta}{\gamma}\left(e^{\gamma(I_{\bar{\mathbf{c}}}-3)} - 1\right).$$

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Weakened artery

$$\overline{\mathcal{W}}(I_{\bar{\mathbf{c}}}) = \frac{(1-d)\eta}{\gamma}\left(e^{\gamma(I_{\bar{\mathbf{c}}}-3)} - 1\right).$$

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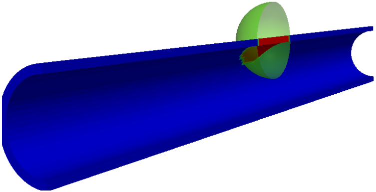
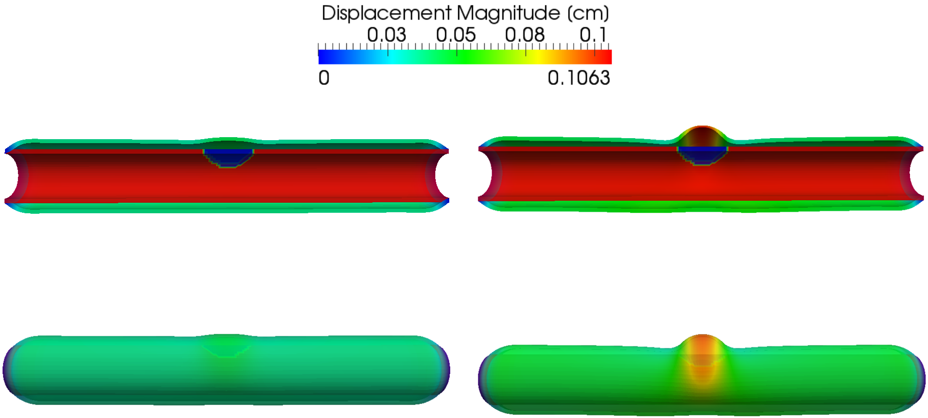
Three levels of weakeninhg

- $(1-d) = 0.7$,
- $(1-d) = 0.3$,
- $(1-d) = 0.15$.



Inflation of a weakened artery



- Spatial variations of the wall thickness in the damaged zone dependent on the level of weakning;
- Higher weakning of the material stiffness induces non local effects on the deformations of the geometry.

Inflation of a weakened artery - Deformations of the damaged zone

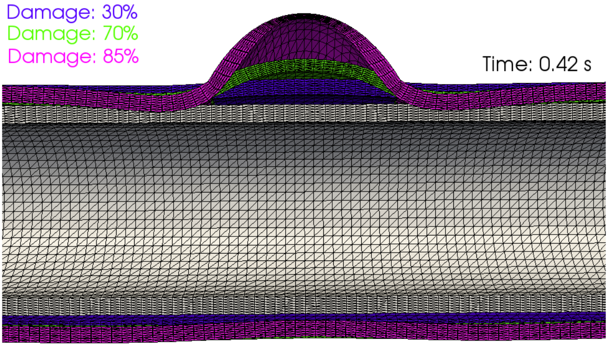
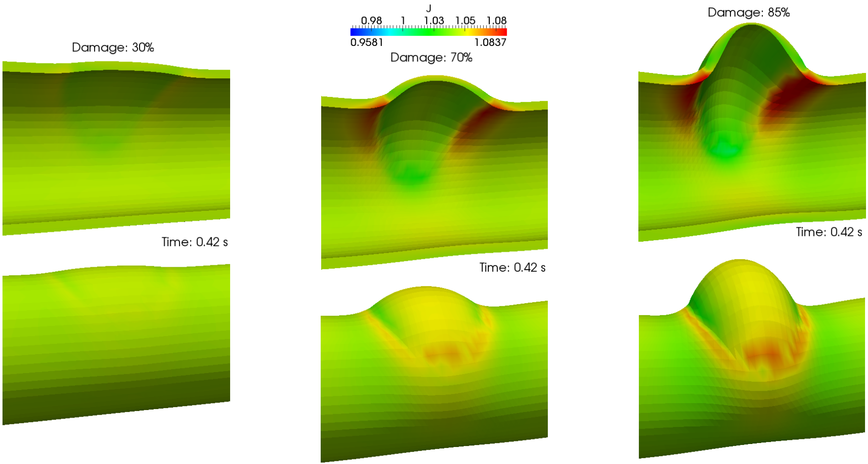


Figure: Deformations, at 6 kPa, of the central portion of the geometry as a function of the weakning level.

At high levels of weakning, even for moderate transmular pressures, the non-local effects become more evident.

Inflation of a weakened artery - Incompressibility constraint



The approximation of the volume-preserving constraint affected by:

- the form of weaking of the tissue that has been considered;
- the level of weaking.

Inflation of a weakened artery - The role of the bulk modulus

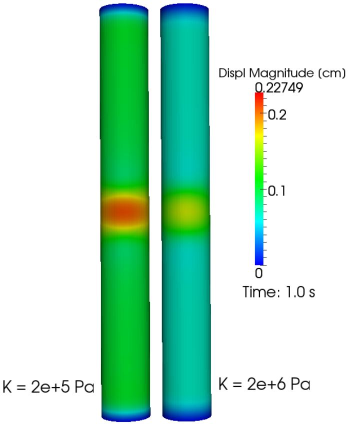
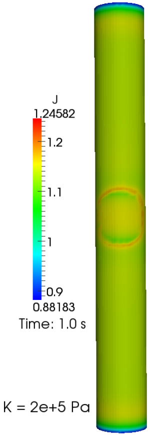


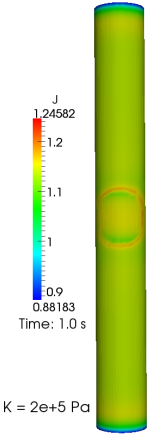
Figure: Displacement magnitude at the transmural pressure of 13.3 kPa.

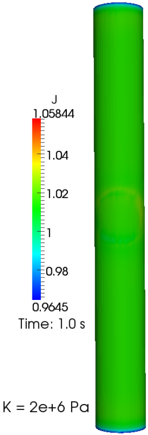
As expected, higher bulk moduli induce smaller displacements.

Inflation of a weakened artery - The role of the bulk modulus



Inflation of a weakened artery - The role of the bulk modulus





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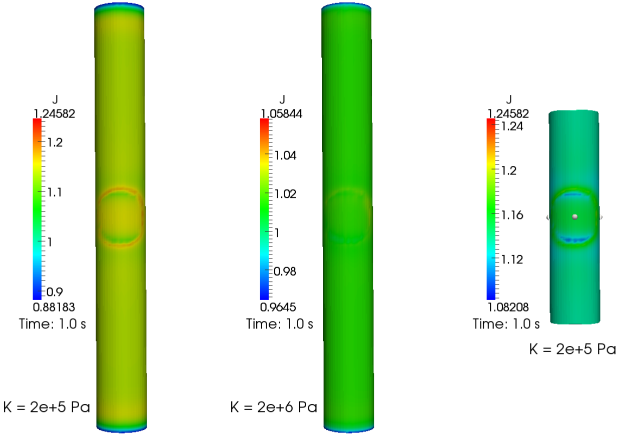
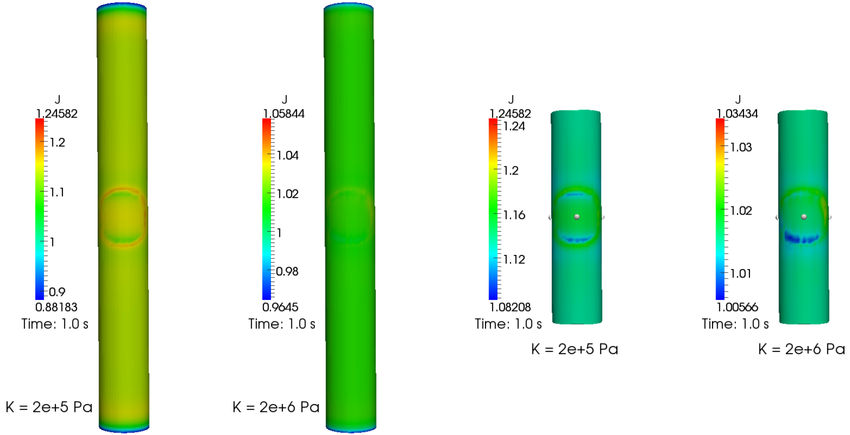
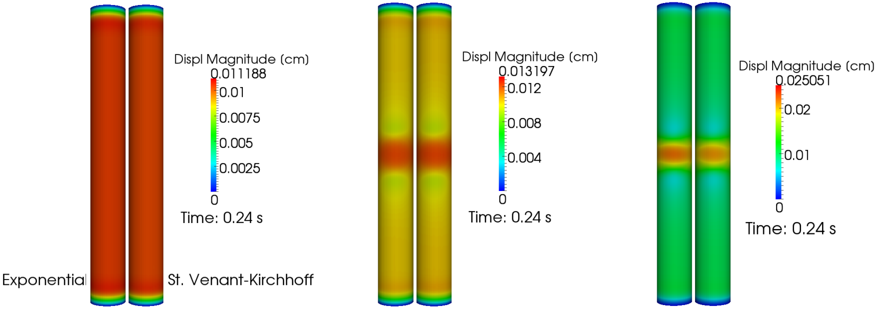


Inflation of a weakened artery - The role of the bulk modulus



In the case of weakened material, the effects of high bulk moduli on the constraint $J = 1$ are more relevant than in the healthy case.

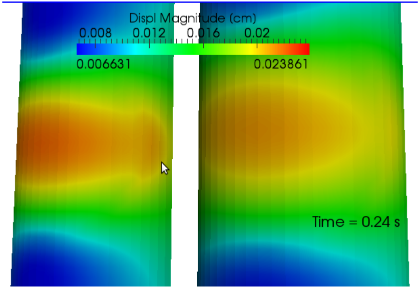
Inflation of weakened tissue - Influence of the structural law



(a) Healthy case (b) Weakning level: 30% (c) Weakning level: 70%

Figure: Comparison of the displacement fields obtained using two different laws.

Inflation of weakened tissue - Influence of the structural law (2)



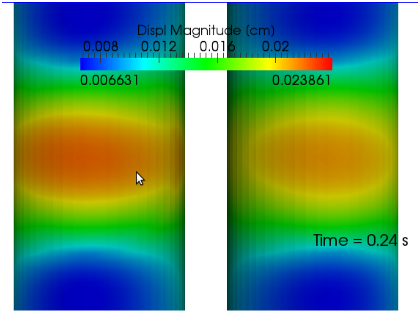


Figure: Particular of the displacement field of the geometry in the central portion.

Cocnlusions and further developments

Conclusions:

- The incompressibility constraint can better approximated modifying the bulk modulus but leading to smaller displacements;
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- For high levels of weakning, thinning and non-local effects become prominent.

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Ongoing work and further developments:

- Simulate the coupled fluid-structure system;
- Consider anisotropic models for the arterial tissue (e.g taking into account the collagen fibers);
- Consider damage models to describe the progressive weakning of the tissue.

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CMCS

cemat IST CENTRO de MATEMÁTICA E APLICAÇÕES

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CSCS Swiss National Supercomputing Centre

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The LifeV community, www.lifev.org