# Modelling cardiac activation

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**Abstract**

The computational modelling of the cardiovascular system aims to aid medical researchers and doctors in developing new methods for diagnosis and therapy. The mathematical description of the mechanics of cardiac contractions and the underlying numerical strategies are still far from being satisfactorily resolved. Moreover, many important physiological details of the dynamics involved in a single heartbeat process are missing as well, even in recent models. The excitation-contraction mechanisms in the cardiac muscle are coordinated by an autonomous electrical activation generated in the sino-atrial node and propagated through the heart wall. This phenomenum, taking place at the subcellular level, creates macroscopic effects which are essential for the cardiac cycle.

Following [1], we contruct a thermodynamical consistent description for a contracting isolated cardiac cell. Contraints arising from the second law of thermodynamics give rise to a constitutive law for the characterization of active deformations

$$\frac{\mathrm{d}\gamma_f}{\mathrm{d}t} = \mathcal{G}\left(\gamma_f, \left[Ca^{2+}\right]\right).$$

We use an active strain approach [2, 3] to describe the myocytes contraction and stresses, where the deformation gradient tensor admits the multiplicative splitting

$$\mathbf{F} = \mathbf{F}_P\mathbf{F}_A = \mathbf{F}_P\left(\mathbf{I} + \gamma_f\mathbf{f}_0\otimes\mathbf{f}_0 + \gamma_s\mathbf{s}_0\otimes\mathbf{s}_0 + \gamma_n\mathbf{n}_0\otimes\mathbf{n}_0\right).$$

The cellular mechanics is cast in the framework of continuum finite elasticity and in particular an incompressible nonlinear hyperelastic models is used to describe the passive structure of the cardiac cell. The cellular excitation phase is described through a two variables model for intracellular Calcium dynamics that consider the Calcium induced-Calcium release phenomenum[5].

This models serve as a playground for a physiological mathematical description of the electromechanical activity of the heart. The contribution of this work is in the framework of the heart integration (HI) project we are carrying on to describe the full physics of the cardiac system.

**Keywords:** Cardiac mechanics, excitation-contraction coupling, active strain formulation

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