Optimal control of non-Newtonian fluids

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Abstract

We consider optimal control problems of non-Newtonian systems governed by quasi-linear, stationary, incompressible Navier-Stokes equations with shear-thinning behaviour viscosity in a two-dimensional or three-dimensional domain. We will first present a mathematical formulation and analysis of a distributed optimal control problem for a viscous and incompressible fluid (see [4, 5]). The goal is to control the system through a distributed mechanical force leading the velocity to a given target field. Both control and state variables are constrained to satisfy a system with shear dependent viscosity which decreases when the shear rate grows. Next, we will show numerical simulations for optimal control problems connected to the Data Assimilation problem. The objective is to minimize the difference between the computed state variable and the data observations registered in part of the domain (see [2, 3]). In this case the control refers to the inlet boundary condition. We considered a 2D domain representing an idealized stenosed vessel. As a test case, we use the model parameters considered in [1] for blood flow. We present a comparative numerical study of Newtonian and non-Newtonian fluids while considering different diameter values for the stenosis.

Keywords: Optimal control , non-Newtonian fluids, shear-thinning, hemodynamics.

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