A new model for blood flow in capillaries

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Abstract

Blood circulation in small vessels has been the subject of a very large literature, starting with the fundamental studies by Poiseuille [15], followed by Murray [13], Landis [9], Fåhraeus [2], Fåhraeus -Lindqvist [3], Krogh [8] and others. Very soon it was clear that blood rheology is very sensitive to the size of the vessel, exhibiting some counter-intuitive behavior when the vessel diameter reaches a range in which the composite nature of blood comes into play. A great influence is also shown by blood composition (namely by the hematocrit) and the way erythrocytes are arranged in the flow, particularly after vessels bifurcations. Clearly the complex mechanics of Red Blood Cells (RBC's) and the way they interact with plasma and among themselves has a fundamental role in microcirculation. The reader is referred to the following recent reviews: Waters et al. [17], particularly dedicated to blood circulation in the heart, Secomb [16], Lee and Smith [10], and Lipowsky [11]. An approach largely used still today for microcirculation is to represent blood as a fluid whose rheological parameters depend on various elements, like, as we said, the blood vessel diameter and the hematocrit. In the present paper we will follow a new approach involving a minimal number of parameters and stemming from the basic principles of mechanics. The main concept inspiring the model is that at the capillary scale it is no longer possible to deal with blood as a "fluid", whatever complicated, since its cellular fraction is going to determine the nature of the motion. Such a distinctive feature of blood flow in capillaries has been emphasized by some authors in the medical literature. From [12] we quote: "...the available fluid mechanical laws cannot be applied for a better understanding of the microcirculation in the living capillaries, and the hemorheology in the microcirculation requires another approach than regularities of the fluid mechanics". Consequently, one has to focus not on the rheological properties of blood as a mixture, but on the motion of its components. Nevertheless, at our knowledge, though the mechanical properties of RBC's and their interaction with blood vessel walls have received a lot of attention (see the already quoted literature and [14]), a rheological model of blood flow in capillaries consistent with the remark above, i.e. on a basis different from classical fluid mechanics, has not yet been produced. Still in [12] the author, in order to stress the importance of the vessel size, refers to Poiseuille's law stating the dependence of pressure

drop on the fourth power of the vessel radius, thus remaining in the domain of classical fluid mechanics, not envisaging which kind of approach has to replace the traditional schemes. In the present study we will adopt an approach in which blood in the capillary is arranged in an idealized sequence of plasma-RBC elements (according to the local value of hematocrit). We will show that inertia is absolutely negligible, thus the motion can be considered as a quasi-steady translation resulting from the equilibrium of two forces: the driving pressure gradient and the drag, mainly originated by the friction of RBC's with the capillary wall. By friction we mean the dissipative process taking place in the highly sheared plasma film interposed between RBC's and the capillary wall, which is responsible for a large pressure drop. Despite the conceptual simplicity of this approach, modeling that kind of flow becomes highly nontrivial for fenestrated capillaries, owing to the progressive loss of plasma. Indeed there are two important consequences of plasma loss:

1. hematocrit increases along the flow, thus increasing the drag;
2. the capillary acts as a semipermeable membrane, giving rise to some osmosis, and this effect is enhanced by the progressive increase of concentration of the molecules not crossing the wall.

In the next section we develop the friction based dynamical model of blood flow in capillaries. Next we will present its main application, namely the determination of the Glomerular Filtration Rate (GFR) in kidneys. Concerning this particular subject, our approach differs from the ones which make ad hoc assumptions on the hydraulic pressure field along the capillaries, taking it uniform (as e.g. in [7]) or linear with a slight slope to be guessed (see the classical paper [1]). In our case the full description of the process follows naturally from well established physical principles. Indeed, it will be enough to calibrate the friction parameters to retrieve the physiological data of GFR. Moreover, the pressure profile can be calculated with good accuracy and it turns out to be in agreement with what previously known (namely, slightly deviating from linearity), confirming that our microcirculation model applies quite well to the important case of glomeruli. Finally, we remark that the translational motion here described is an extreme situation, peculiar to capillaries, and corresponding to full slip at the wall. This circumstance points out that, considering vessels of increasing radius, slip retains some importance at least for sufficiently small vessels, such as arterioles. Such a fact has nontrivial consequences on blood coagulation processes, as it has been shown in [6]. Slip is equally important in blood ultrafiltration by means of hollow fibers devices, as it has been pointed out in [5]. The basic reference for this research is [4].

Keywords: microcirculation , capillaries.

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