A new criterion for the local regularity of a suitable weak solution to the Navier-Stokes equations

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Abstract

Let $\Omega \subset \mathbb{R}^3$ be a domain with $\partial\Omega \in C^2$. We prove the existence of an absolute constant $\varepsilon_\star > 0$, such that if $\{\mathbf{u}, p\}$ is a suitable weak solution to the Navier-Stokes equations in $Q := \Omega\times]0,\infty[$ then for any parabolic cylinder $Q_R \subset Q$ the condition

$$\int_{Q_R} |\mathbf{u}|^3 \,\mathrm{d}x\,\mathrm{d}t \,\leq\, R^2\varepsilon_\star$$

implies

$$\mathbf{u} \quad \textit{is Hölder continuous on} \quad \overline{Q}_{R/2}.$$