

# The general Oberbeck-Boussinesq problem with feedbacks on the body force and heat capacity

Hermenegildo Borges de Oliveira  
*Faculdade de Ciências e Tecnologia*  
*Universidade do Algarve, Portugal*  
*holivei@ualg.pt*

## Abstract

This talk is based in works developed in collaboration with S.N. Antontsev and J.I. Díaz on mathematical models in the thermodynamics of heat-conducting fluid problems. We start by considering the Oberbeck-Boussinesq equations which govern the flow of linearly viscous, homogeneous, incompressible fluids with neither inner body forces nor heat production. The corresponding system of equations is considered with nonlinear thermal diffusion and temperature-depending viscosity. Then, we modify this problem by considering the forces field, in the momentum equation, depending not only on the temperature, but also on the own velocity. The modification affects also the energy equation, where an extra term is added to its right-hand side. This new term depends on the temperature and velocity and it can account for a certain type of energy source in the fluid. Thus, the original Oberbeck-Boussinesq equations together with these modifications origin a new problem. After, we define the notion of solutions of this new problem we shall work with and prove the existence of that solutions. This proof is carried out by adapting the Galerkin method to prove the existence of a velocity field and using known results on nonlinear diffusion equations to prove the existence of a temperature. Restricting us to the dimension 2, we prove also the obtained solutions are unique. In the second and main part of the talk, we use important assumptions on the forces field and on the extra term added to the energy equation to establish some results related with the qualitative properties of that solutions. We show that, contrary to the original Oberbeck-Boussinesq problem, a pair of solutions extinct in a finite time separately. We prove also other properties for the temperature alone, and which are related with the finite speed of propagations and the waiting time properties.

**Keywords:** Oberbeck-Boussinesq, nonlinear thermal diffusion, temperature-depending viscosity, existence, uniqueness, extinction in time, finite speed of propagations, waiting time.

## References

- [1] S. N. Antontsev and H. B. de Oliveira, The general Oberbeck-Boussinesq problem with feedback body forces and heat capacity. *In preparation*.
- [2] S. N. Antontsev, J. I. Díaz and H. B. de Oliveira, Stopping a viscous fluid by a feedback dissipative field: thermal effects without phase changing, *Progr. Nonlinear Differential Equations Appl.* **61**, (2005), 1-14.
- [3] S. N. Antontsev, J. I. Díaz and S. I. Shmarev, *Energy methods for free boundary problems*, Progr. Nonlinear Differential Equations Appl. **48**, Birkhäuser, 2002.