Influence of wall roughness on the slip behavior of viscous fluids

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Abstract

A proper choice of boundary conditions plays a significant role in the problems studied in continuum fluid dynamics. In many theoretical studies as well as numerical experiments, the standard well-accepted hypothesis states that a viscous fluid adheres completely to the boundary of the physical domain provided the latter is impermeable. If $\mathbf{u} = \mathbf{u}(t, x)$ is the Eulerian velocity of the fluid at a time t and a spatial position $x \in \Omega \subset R^3$, impermeability of the boundary $\partial\Omega$ means that

$$\mathbf{u}(t, x) \cdot \mathbf{n}(x) = 0 \text{ for any } x \in \partial\Omega, \tag{1}$$

where $\mathbf{n}$ stands for the outer normal vector, while complete adherence can be formulated in terms of the no-slip boundary condition

$$\mathbf{u}(t, x) = 0 \text{ for any } x \in \partial\Omega. \tag{2}$$

Recently, there have been several attempts to give rigorous mathematical justification of (eq2) based on the concept of rougous boundary, see [4]. The main idea is to assume that the " real " boundary is never perfectly smooth but contains microscopic asperities of the size significantly smaller than the characteristic length scale of the flow. The " ideal" domain physical domain Ω is being replaced by a family $\{\Omega_\epsilon\}_{\epsilon>0}$ of "rough" domains, where the parameter $\epsilon > 0$ stands for the amplitude of asperities. Assuming only the impermeability condition (eq1) on the $\partial\Omega_\epsilon$ one can show that the stronger no-stick boundary conditions must be imposed for the limit problem when $\Omega_\epsilon \to \Omega$ in some sense, provided the distribution of asperities is uniform, more specifically spatially periodic, and "non-degenerate" see [1]. In case when rugosity is "degenerate" in one direction, the limit case satisfies an intermediate boundary conditions - a weak " partial slip" boundary condition see [2] or more general directional Navier slip condition see [3].

Keywords: Navier-Stokes equations, no-slip boundary conditions, slip boundary conditions, partial slip, Navier slip conditions.

References

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