The geometrical multiscale modelling of the cardiovascular system: coupling 3D and 1D FSI models

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Abstract

Three-dimensional (3D) simulations of blood flow provide detailed information important for the comprehension of the cardiovascular system. These consist in coupling the 3D Navier-Stokes equations with a model for the vessel wall structure [1]. Due to the computational cost of fully 3D fluid-structure interaction (FSI) problems and to the complexity of the cardiovascular system, these models can be applied only to truncated regions of interest [2]. Following the geometrical multiscale modelling of the cardiovascular system, the remaining parts can be accounting resorting to reduced models [3], one-dimensional (1D) or zero-dimensional (0D). In particular, the 1D models describe very well the wave propagation nature of blood flow and coupled with the 3D model act as proper absorbing boundary conditions. We address the coupling of 3D and 1D FSI models [4]. The difficulty of this coupling lays in putting together such different models, namely it is not evident which conditions to impose at the coupling interface. We study the stability of such coupling, bringing forth proper matching conditions between the models. This is carried out through a reformulation of the Navier-Stokes equations applied on the 3D model [5]. Moreover we consider different structural models for the vessel wall on the 3D domain and study their influence on the coupling. Different coupling strategies are discussed, including implicit and explicit couplings. We present several numerical results, including an anatomically 3D realistic compliant model of a human carotid bifurcation coupled with reduced models at the downstream sections.

Keywords: Geometrical multiscale modelling, fluid-structure interaction, 3D-1D FSI coupling, energy estimate.

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