On a coupled free boundary problem for a piezoviscous fluid in thin film

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Abstract

The aim of this talk is to state an unconditional existence result for elastohydrodynamic piezoviscous lubrication problems with Elrod-Adams model of cavitation. In many practical applications the fluids involved have a more complex behaviour as the Newtonian ones. For example in lubrication problems, in which the fluid is exposed to very high stresses, the viscosity is no longer constant. Usual models introduce a pressure-viscosity behaviour, as the Barus relation $\mu(p) = \mu_0 \, e^{\alpha p}$, with μ_0, α positive constants. In such models, the constants are often determined experimentally. A second feature of the flow considered in this problem is the anisotropic aspect ratio of the flow. Thus, three-dimensional Navier-Stokes system is replaced by the two-dimensional Reynolds equation on the pressure p:

$$\operatorname{div}\left(\frac{1}{\mu} h^3 \nabla p\right) - \lambda_0 u \frac{\partial h}{\partial x} = 0,$$

where h is the small dimension of the domain with respect to the others, u is the tangential velocity on one side of the boundary, and λ_0 is a constant. The third feature taken into account is the free boundary phenomenon called cavitation, which can be modelled in two different ways:

- a variational inequality, which corresponds to the minimization of a functional under the condition on the pressure $p \geq 0$;
- a pressure-saturation model, which considers that the cavitation zone is characterized by a constant pressure and an homogeneous blend of air and fluid, and introduces a function θ, corresponding to the local proportion of the fluid.

At last, due to the high pressures, the upper surface of the domain is deformed. This phenomenon is modelled by the following relation $h = h_0 + k \star p$, where k

is a given kernel. Thus, using the second model, the problem can be written in the following form:

$$\mathrm{div}\left(\frac{h^3(p)}{\mu(p)}\nabla p\right) = 6u\frac{\partial(\theta(p)\,h(p))}{\partial x}, \qquad \text{with } \theta \in H(p) \qquad \text{(Heaviside function).}$$

For both models of cavitation, conditional existence results have been proved ([3], [2]). However, the hypothesis made on the physical parameters does not seem to be verified in the experiments. In the variational inequality case, an unconditional existence result have been proved ([1]) under a realistic hypothesis on the asymptotic behaviour of the viscosity. The aim of this work is to obtain a unconditional existence result with the pressure-saturation problem. After presenting the model, the mathematical approach of the problem consists in introducing its regularization for which the existence is trivial. It remains to prove that the regularized problem converges to the real one. In a first part, some classical estimates are obtained. However, without conditions on the physical parameters, it is not enough to conclude. Thus, assuming a realistic hypothesis on the asymptotic behaviour of the viscosity, additional estimates are proved.

Keywords: Elastohydrodynamics, free boundary, lubrication, two-phase flow, partial differential equations.

References

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