Lifting solutions of quasilinear convection dominated problems

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Abstract

The steady state of the quasilinear convection-diffusion-reaction equation

$$u_t - \nabla(D(u)\nabla u) + b(u)\nabla u + c(u) = 0 \qquad (1)$$

is studied.

Depending on the ratio between convection and diffusion coefficients, equation (1) ranges from parabolic to almost hyperbolic. From a numerical point of view two main difficulties can arise related with the existence of layers and/or the non smoothness of the coefficients.

In this talk we present a new numerical method for solving the steady state equation associated with (1). This method is based on the idea of solving an associated modified problem whose solution corresponds to a lifting of u. The method introduced here avoids an a priori knowledge of the layer(s) location and allows an efficient handling of the lack of the smoothness of the coefficients. Numerical results are included.

Keywords: Convection dominated problem, layers, nonuniform meshes.

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