Numerical Simulations of Clot Growth Evolution in Small Arteries Using a Reduced Mathematical Model

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Abstract

The ability of the body to control the blood flow following vascular injury is crucial for survival. Blood coagulation is an extremely complex biological process that causes blood to form the clots, preventing the blood loss, followed by their dissolution and the subsequent repair of the injured tissue. This process involves complex interactions among multiple molecular and cellular components in the blood and vessel wall, and it is also influenced by the flow of blood.

Disorders of the blood coagulation system, due to the disturbance of appropriate coagulation process components, can increase the risk of bleeding or obstructive clotting.

Mathematical modeling of blood coagulation and fibrinolysis processes is a way of conceptualizing and understanding this complicated system. It helps to identify those regions of the vascular system which are more exposed to the formation of thrombotic plaques and possible rupture in stenosed arteries. A good model should be simple enough to be used for numerical simulations, and at the same time should be able to capture the main features of the complex process to allow for its better understanding.

Since the blood coagulation process includes a large number of coagulation factors, the mathematical model becomes very difficult due to a great amount of data to be processed. The idea behind this work is to concentrate on the propagation phase of the blood coagulation process with primary focus on the clot growth and its evolution within the flow, assuming that the initial and amplification phases of the cell-based coagulation model are quite fast compared to the whole process and do not influence the clot evolution.

The effect of blood slip is introduced at the vessel wall emphasizing an extra supply of activated platelets to the clotting site. The expectations are that such contribution could be dominant, resulting in the acceleration of thrombin production and eventually of the whole clot progression. Such model will have the capacity to predict effects of specific perturbations in the hemostatic system that cannot be undertaken by laboratory tests.

Numerical results of ongoing work in 2D case, based on the solution of the system of RAD equations coupled to the rheological model describing the blood flow, will be presented. Evolution of clot growth and the importance of the blood flow on its formation will be discussed and the concentration behavior of coagulation factors will be investigated in the injury site of the vessel wall.

Keywords: Blood coagulation, mathematical modeling, reaction-advection-diffussion equations, numerical simulation

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