Isogeometric Analysis and reduced models for Fluid–Structure Interactions problems in Haemodynamics

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Abstract

The simulation of blood flows in the cardiovascular system in physiological or pathological conditions requires the numerical approximation of Fluid–Structure Interaction (FSI) problems [1, 3]. FSI models in Haemodynamics couple the Navier–Stokes equations representing the blood flows with structural models describing the mechanical response of the deformable walls of the arteries. Such FSI models are in general extremely complex, both from the mathematical and numerical points of view, since defined in non–rigid domains moving consequently to the fluid and solid interactions. Therefore, the numerical simulation of FSI models is generally demanding in terms of computational resources and requires the definition and use of suitable numerical methods. In this context, efficient and reduced FSI models can be introduced with in order to reduce the complexity of the fully coupled problem.

With this aim, thin membrane models can be adopted for the representation of the mechanical response of the arteries [4], often leading to high order Partial Differential Equations (PDEs) defined on the interfaces between the fluid and arterial tissues. In addition, such simplified structural models can be rewritten as generalized boundary conditions for the Navier–Stokes equations describing the blood flow [4, 5]. In this manner, it is possible to define reduced FSI models as generalized Navier–Stokes equations defined

in fixed domains for which the structural response is embedded in suitable generalized boundary conditions [4]. As consequence, the computational procedure is significantly simplified since the reduced models are defined in fixed computational domains and written only in terms of the primitive variables of the Fluid Dynamics equations. Conversely, such generalized boundary conditions involve derivatives of the velocity field, eventually high order derivatives. When considering the Finite Elements method, such generalized boundary conditions may be inaccurately represented since the velocity field and the geometry are approximated by means of globally continuous basis functions.

Isogeometric Analysis (IGA) [2] is a computational methodology based on the isogeometric paradigm for which the same basis functions used to represent the geometry are then also used for the approximation of the solutions of the PDEs. In this manner, the exact geometric representation is encapsulated in the numerical approximation of the PDEs and, in the case that NURBS basis functions are used, high order PDEs can be solved with standard Galerkin formulations. The efficiency of IGA for the solution of full FSI problems is highlighted e.g. in [1].

In this work we consider IGA for the numerical approximation of reduced FSI models. In particular, we highlight the efficacy and accuracy of the numerical procedure for the representation of the generalized boundary conditions, especially when high order PDEs are considered for modeling the mechanical response of the arterial walls in the reduced FSI models.

Keywords: Isogeometric Analysis, Fluid–Structure Interaction, Haemodynamics.

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